On the Origin of Current Scaling in the Density Limit

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Outline

- Density Limit ⇔ Transport Phenomena

  Shear Layer Collapse

- Why? → A Model:

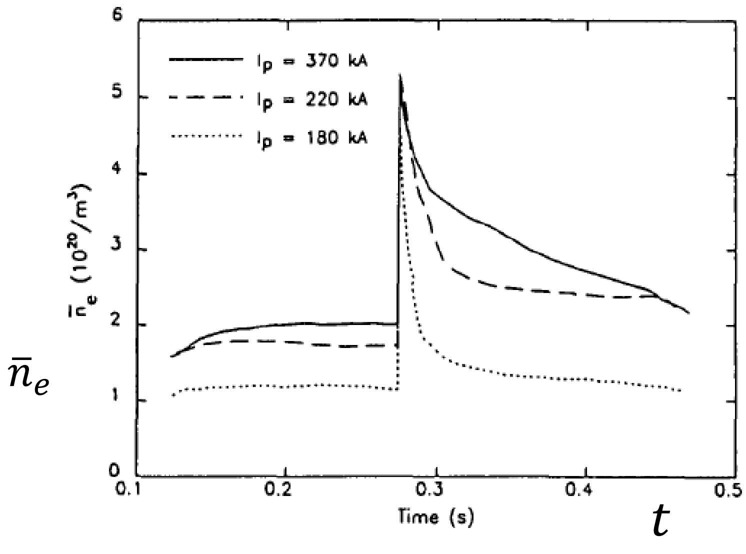
  Adiabatic → Hydrodynamic Transition

Desperately seeking Greenwald → Origin of Current Scaling?!

- Neoclassical Dielectric and Zonal Flow Inertia
- Resolving the Collisionality Issue – Plateau Regime
- Other Implications – L-H Transition
- Beyond Tokamaks
- Conclusions

A Look at Density Limit Phenomenology

- Starting Point: Edge Particle Transport is crucial
  - ‘Disruptive’ scenarios secondary outcome, largely consequence of edge cooling, following fueling vs. increased particle transport
  - $\bar{n}_g$ reflects fundamental limit imposed by particle transport
- A Classic Experiment (Greenwald, et. al.)



(Alcator C)

- Density decays without disruption after shallow pellet injection
- $\bar{n}$ asymptote scales with I_p
- Density limit enforced by transport-induced relaxation
- Relaxation rate not studied

Synthesis of the Experiments

[Y. Xu, et. al.; Schmidt, et. al., Hong and Tynan, et. al.; Tynan, et. al.]

- Edge Shear layer collapse and turbulence and D (particle transport) rise as $\frac{\bar{n}}{\bar{n}_G} \to 1$.

  → Key microphysics of density limit !?

- ZF collapse as $\alpha = \frac{k_{||}^2 v_{th}^2}{|\omega| \nu_e}$ drops from $\alpha > 1$ to $\alpha < 1$.

  → Effect on production

- Degradation in particle confinement at density limit in L-mode is due to breakdown of self-regulation by zonal flow

- Note that β in these experiments is too small for conventional Resistive Ballooning Modes (RBM) explanation.

⇨ How reconcile all these with our understanding of drift wave-zonal flow physics?

The Key Questions

- What physics governs shear layer collapse (or maintanance) at high density?

⬄ 'Inverse process' of familar L→H transition !?

i.e. L→H : { shear layer → barrier; turbulence }

Density Limit: strong turbulence ← { shear layer, turbulence }

➔ In particular, what is the fate of shear flow for hydrodynamic electrons: $k_{\parallel}^2 V_{th}^2 / \omega \nu < 1$?

A Theory of Shear Layer Collapse

Reduced Model (from H-W)

$$l_{mix} = \frac{l_0}{\left(1 + \frac{(l_0 \nabla u)^2}{\varepsilon}\right)^{\delta}} \rightarrow l_0$$

$$\partial_t n = -\partial_x \Gamma_n + D_0 \nabla_x^2 n$$

(density)

N.B.: Encompasses 'predator-prey' model

$$\partial_t u = -\partial_x \Pi + \mu_0 \nabla_x^2 u$$

(vorticity/shear[zonal])

$$\partial_t \varepsilon + \partial_x \Gamma_\varepsilon = -(\Gamma_n - \Pi)(\partial_x n - \partial_x u) - \varepsilon^{\frac{3}{2}} + P$$

(fluctuation potential enstrophy ~ I)

- Fluxes:

Γ_n → Particle flux $\langle \tilde{V}_x \tilde{n} \rangle$

Π → Vorticity flux $\langle \tilde{V}_x \nabla^2 \tilde{\phi} \rangle = -\partial_x \langle \tilde{V}_x \tilde{V}_y \rangle$ (Taylor, 1915)

⇕

Reynolds Force

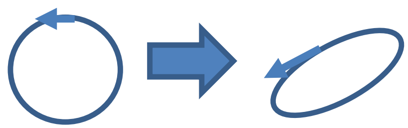
Γ_ε → turbulence spreading, $\langle \tilde{V}_x \tilde{\varepsilon} \rangle$ → triad interactions

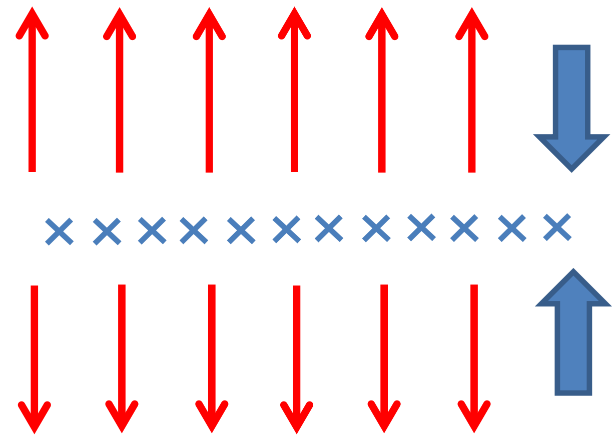
Step Back: Zonal Flows Ubiquitous! Why?

- Direct proportionality of wave group velocity and wave energy density flux

  to Reynolds stress ←→ spectral correlation $\langle k_x k_y \rangle$

  Causality ←→ Eddy Tilting

  i.e. 

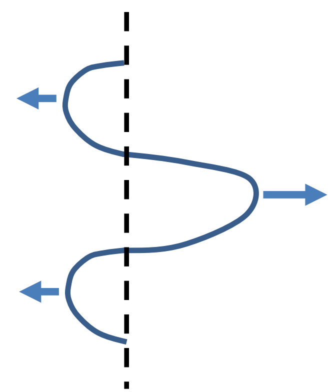
  

  $\omega_k = -\beta\, k_x/k_\perp^2$: (Rossby)

➔ $V_{g,y} = 2\beta\, k_x k_y/(k_\perp^2)^2$

➔ $\langle \tilde{V}_y \tilde{V}_x \rangle = -\sum_k k_x k_y |\phi_k|^2$

  So: $V_g > 0\ (\beta > 0)$ ←→ $k_x k_y > 0$ ➔ $\langle \tilde{V}_y \tilde{V}_x \rangle < 0$



- Outgoing waves generate a flow convergence! ➔ Shear layer spin-up

But NOT for hydro convective cells:

- $\omega_r = \left[\frac{|\omega_{*e}|\hat{\alpha}}{2k_\perp^2\rho_s^2}\right]^{1/2}$ → for convective cell of H-W
- $V_{gr} = -\frac{2k_r\rho_s^2}{k_\perp^2\rho_s^2}\,\omega_r$ ←??→ $\langle\tilde{V}_r\tilde{V}_\theta\rangle = -\langle k_r k_\theta\rangle$; direct link broken!

→ Energy flux NOT simply proportional to Momentum flux ➔

↓

→ Eddy tilting ($\langle k_r k_\theta\rangle$) does not arise as direct consequence of causality

➔ ZF generation not 'natural' outcome in hydro regime!

➔ Physical picture of shear flow collapse emerges

Scaling of transport fluxes with α (adiabaticity parameter)

Plasma Response	Adiabatic (α >>1)	Hydrodynamic (α <<1)
Particle Flux Γ	$\Gamma_{\mathrm{adia}} \sim \frac{1}{\alpha}$	$\Gamma_{hydro} \sim \frac{1}{\sqrt{\alpha}}$
Turbulent Viscosity χ	$\chi_{adia} \sim \frac{1}{\alpha}$	$\chi_{hydro} \sim \frac{1}{\sqrt{\alpha}}$
Residual stress Π^{res}	$\Pi^{res}{}_{\mathrm{adia}} \sim -\frac{1}{\alpha}$	$\Pi^{res}{}_{\mathrm{hydro}} \sim -\sqrt{\alpha}$
$\frac{\Pi^{\mathrm{res}}}{\chi}$ = Vorticity Gradient	α^0	α^1

$\Gamma_n, \chi_y \uparrow$ and $\Pi^{\mathrm{res}} \downarrow$ as the electron response passes from adiabatic (α >1) to hydrodynamic (α <1)

α <1 → <u>weak flow production</u>

- Mean vorticity gradient ∇u (i.e. ZF strength) proportional to $\alpha \ll 1$ for convective cells.
- Weak ZF formation for $\alpha \ll 1$ → weak regulation of turbulence and enhancement of particle transport and turbulence.

Desperately Seeking Greenwald

- What of Current Scaling? – Key Question!
- Collisionality – Screening for the Plateau Regime?!
- Tokamaks, RFP, Stellarators

What of the Current Scaling?

- Obvious question: How does shear layer collapse scenario connect to Greenwald scaling $\bar{n} \sim I_p$? i.e. current favorable!
- Key physics: shear/zonal flow response to drive is 'screened' by dielectric – both classical and neoclassical → two scales

i.e.
- $\epsilon_{neo} = 1 + 4\pi\rho c^2/B_\theta^2$ (banana regime)
- ρ_θ as screening length
- effective ZF inertia lower for larger I_p

ZF – modes of minimum { inertia, damping, transport }

Current Scaling, cont'd

- Shear flow drive:

emission from 'drift-mode' interaction

production

$$\frac{d}{dt}\left[\langle\left(\frac{e\phi}{T}\right)^2\rangle_{ZF}\right] \approx \frac{\sum_k \left|S_{k,q}\right|^2 \tau_{c_{k,q}}}{|\epsilon_{neo}(q)|^2}$$

neoclassical response

  - Production ←→ beat drive (polarization)
  - Response (neoclassical)

- Rosenbluth-Hinton '97 et seq (banana regime)

Increasing I_p decreases ρ_θ, can off-set weaker ZF drive

$$\left(\frac{e\hat{\phi}}{T}\right)_{ZF} \approx \frac{S_{k,q}}{\left(1 + 1.16\frac{(q(r))^2}{\epsilon^{1/2}}\right) q_r^2 \rho_i^2}$$

classical

neo

zonal wave #

Current Scaling, cont'd

$$\left(\tilde{V}_E'\right)_Z \approx \frac{S_{k,q}}{\left[\rho_i^2 + 1.6\epsilon_T^{\frac{3}{2}}\rho_{\theta i}^2\right]} \sim P\,\frac{\left(\frac{e\phi}{T}\right)^2}{\rho_{\theta i}^2} \sim B_\theta^2\,P\,\left(\frac{e\phi}{T}\right)_{DW}^2$$

production → $P \sim n^{-\alpha}$

- Higher current strengthens ZF shear, for fixed drive
- Can support shear layer vs weaker production
- Collisionality? – Edge of interest!?

Screening in the Plateau Regime!?

$$\left(\frac{\phi_k(\infty)}{\phi_k(0)}\right)^{ZF} = \frac{\epsilon^2/q(r)^2}{\left(\epsilon/q(r)\right)^2 + L} \approx \frac{\epsilon^2/q(r)^2}{L} = \frac{1}{L}\left(\frac{B_\theta}{B_T}\right)^2$$

$$L = \frac{3}{2}\int_0^{1-\epsilon} d\lambda \frac{\int d\theta}{2\pi} h^2 \rho \approx 1 - \frac{4}{3\pi}(2\epsilon)^{3/2}$$

- Favorable I_p scaling of time asymptotic RH response persists in plateau regime. Robust trend.
- Compare to Banana ($L = 1$);

$$\left(\frac{\phi_k(\infty)}{\phi_k(0)}\right)^{ZF} = \left(\frac{B_\theta}{B_T}\right)^2$$

Current scaling but smaller ratio

Summary re Collisionality

- Banana(RH) $\nu_{ii} < \omega_{bi} < \omega_{Ti}$ $\quad \dfrac{\phi_k(\infty)}{\phi_k(0)} = \left(\dfrac{B_\theta}{B_T}\right)^2 \sim I_p^2$

- Plateau $\omega_{bi} < \nu_{ii} < \omega_{Ti}$ $\quad \dfrac{\phi_k(\infty)}{\phi_k(0)} = \left(\dfrac{B_\theta}{B_T}\right)^2 \dfrac{1}{L}$ $\quad L < 1$

- Pfirsch-Schluter $\omega_{bi} < \omega_{Ti} < \nu_{ii}$ $\quad \dfrac{\phi_k(\infty)}{\phi_k(0)} = 1$ $\quad \rho_{sc} = \rho_i$

➔ GAM can still exhibit favorable trend with I_p

Related Points

- Effective inertia of zonal flows minimal in P-S
- Optimal for { triggering of edge ZF at L→H; maintaining ZF in H-mode }
- Principle neoclassical effect on $V_{E\times B}$ is enhanced inertia (polarization)
- Often quoted $(1+2q^2)$ factor applies to mass flow, not $E\times B$ → Irrelevant!

General Conclusions

- Transport is fundamental to density limit. Cooling, etc. drive secondary phenomena.
- Shear layer collapse occurs as transport bifurcation from DW-ZF turbulence to convective cells, approaching density limit.
- Trends of Greenwald scaling follow from neoclassical zonal flow response.

Back-Up

What of other Donuts? Pretzels?

- All devices exhibit edge shear layer in L-mode and many similar fluctuation properties (Carreras, Hidalgo et. al.)
- RFP ~ Cylinder ➔ 'neoclassical' effects ignorable

But:

- RFP exhibits Greenwald scaling $n \sim I_p$!
- Classical ZF response → ρ_i , but ρ_i set by current in RFP i.e. $\rho_i = \rho_{\theta i}$
- Stronger ZF shear at higher current!
- Consistent with collisional regimes

What of Stellarator?

- Several attempts to ‘translate’ Greenwald scaling into stellarator (‘magnetic geometry thinking): B_θ $\rightarrow$ iota, shear, with dubious outcomes.
- If ZF screening crucial, better ask: “What length scale appears in Z.F. response for stellarator?”
- Sugama-Watanabe: Principlal correction to classical screening is contribution from helically trapped particle (analysis for LHD).
- Can regard ZF screening length as effectively classical i.e. ρ_i

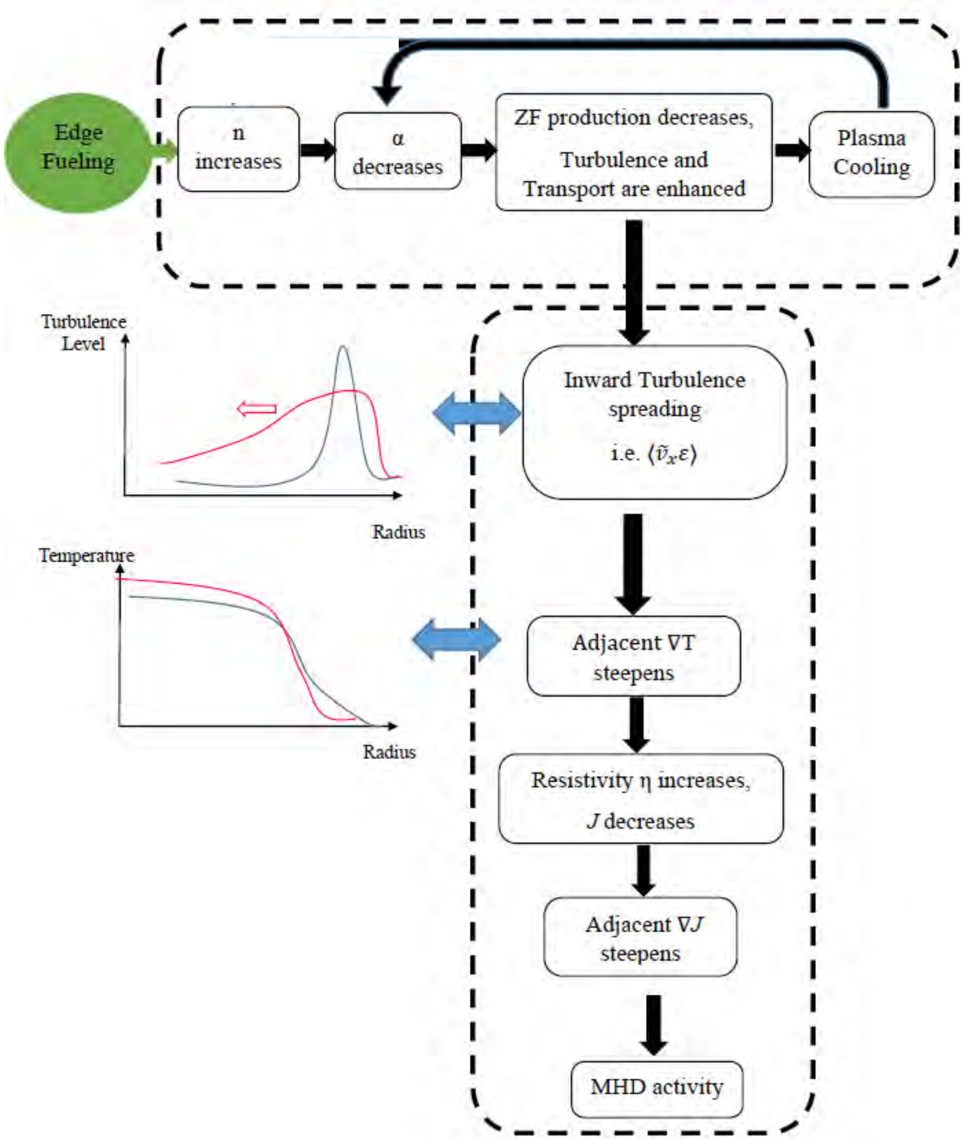
What of Stellarator?, cont'd

- <u>No</u> obvious length scale emerges, other than ρ_i

➔Begs: Will optimized stellarator have higher density limit due more robust edge shear layer?, since $\rho_{screen} \sim \rho_i$?

➔Issue remains open

The Big Picture



Production ↓ → Cooling ↑
Feedback Loop

→ post-collapse intensity increase

→ inward spreading

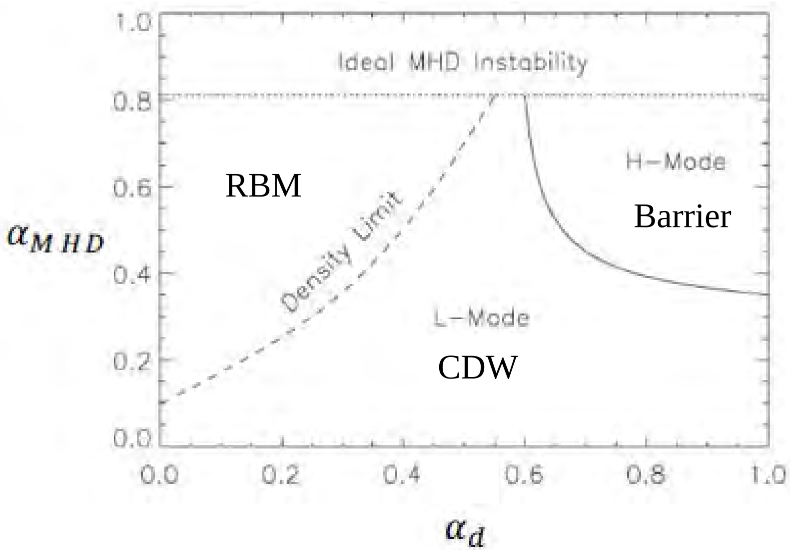
→ turbulence spreading 'transmits' edge cooling to low q resonance

Key: $[r_{sep} - r_q]$ vs $(D\tau_c)^{1/2}$

A Developing Story

From Linear Zoology to Self-Regulation and its Breakdown

(Drake and Rogers, PRL, 1998)



- $\alpha_{MHD} = -\frac{Rq^2 d\beta}{dr} \rightarrow \nabla P$ and ballooning drive to explain the phenomenon of density limit.
- Invokes yet another linear instability of RBM.
- What about density limit phenomenon in plasmas with a low β?

(Hajjar et al., PoP, 2018)

State	Electrons	Turbulence Regulation
Base State - *L*-mode	Adiabatic or Collisionless $\alpha > 1$	Secondary modes (ZFs and GAMs)
H-mode	Irrelevant	Mean ExB shear ∇Pi/n
Degraded particle confinement (Density Limit)	Hydrodynamic $\alpha < 1$	None - ZF collapse due weak production for $\alpha < 1$

Secondary modes and states of particle confinement

L-mode: Turbulence is *regulated* by shear flows, but not suppressed.
H-mode: *Mean ExB* shear $\leftrightarrow \nabla p_i$ suppresses turbulence and transport.
Approaching Density Limit: High levels of turbulence and particle transport, as shear flows collapse.

i.e. Shear Flow: Density Limit > | < L-mode > | < H-mode

Weak (none) | Modest | Strong Mean

Hasegawa-Wakatani Model

Hasegawa-Wakatani for Collisional DWT:

$$\frac{dn}{dt} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_\parallel^2(\phi - n) + D_0\nabla^2 n$$

$$\frac{d\nabla^2\phi}{dt} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_\parallel^2(\phi - n) + \mu_0\nabla^2(\nabla^2\phi)$$

$$\alpha = \frac{k_{||}^2 v_{th}^2}{|\omega|\nu_{ei}}$$

Fluctuations

$$\partial_t\tilde{n} + \tilde{v}_x.\nabla\bar{n} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_\parallel^2(\tilde{\phi} - \tilde{n}) - \{\tilde{\phi}, \tilde{n}\} + D_0\nabla^2\tilde{n}$$

$$\partial_t\nabla^2\tilde{\phi} + \tilde{v}_x.\nabla\overline{\nabla^2\phi} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_\parallel^2(\tilde{\phi} - \tilde{n}) - \{\tilde{\phi}, \nabla^2\tilde{\phi}\} + \mu_0\nabla^2(\nabla^2\tilde{\phi})$$

Mean Fields

$$\partial_t\bar{n} = -\partial_x\langle\tilde{V}_x\tilde{n}\rangle + D_0\bar{\nabla}_x^2\bar{n}$$

$$\partial_t\overline{\nabla_x^2\phi} = -\partial_x\langle\tilde{V}_x\nabla^2\tilde{\phi}\rangle + \mu_0\nabla_x^2\overline{\nabla_x^2\phi}$$

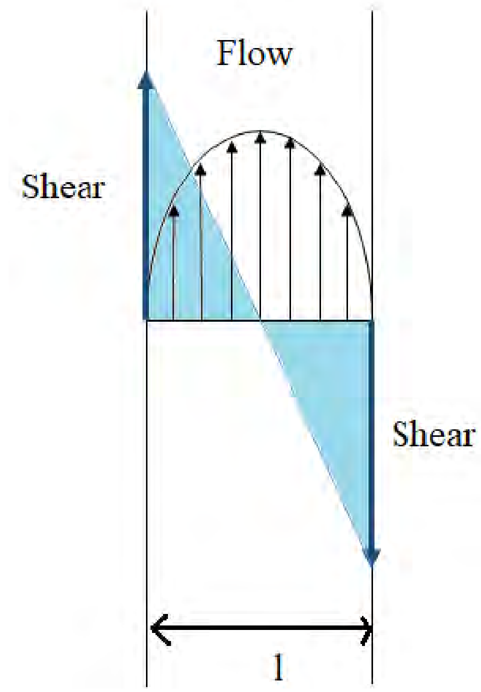
For neoclassical mean field evolution

$$\rho_i^2 \to \rho_{eff}^2 \approx \rho_{\theta i}^2, \ldots$$

Some Theoretical Matters

Physics of Vorticity Gradient ?!

- ∇u, not flow shear, is natural flow order parameter
- [Jump in flow shear, over scale l] = [∇u, over scale l]
- Vorticity gradient prevents local alignment of eddy or mode with shear
- $\Pi = 0 \quad \rightarrow \quad \nabla u \ \sim \ \Pi^{res}/x_y$
- Standard interpretation: Enhanced 'drift wave elasticity' → ∇u converts turbulence to waves, so reducing mixing.



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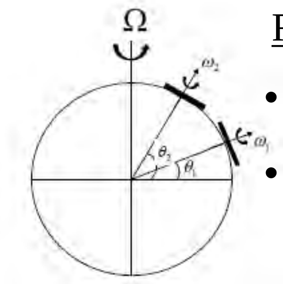
ZF Collapse ←→ PV Conservation and PV Mixing?

How reconcile?

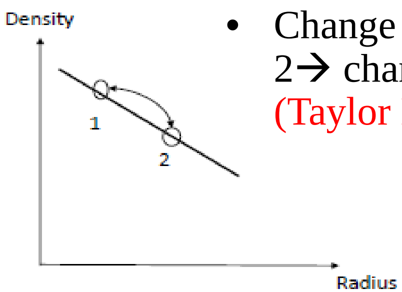


Rossby waves:

- $PV = \nabla^2\phi + \beta y$ is conserved from θ_1 to θ_2.
- Total vorticity $2\vec{\Omega} + \vec{\omega}$ frozen in→ Change in mean vorticity Ω leads to change in local vorticity ω → Flow generation (Taylor's ID)

Drift waves:

- In HW, $q = \ln n - \nabla^2\phi = \ln n_0 + h + \tilde{\phi} - \nabla^2\phi$ conserved along the line of density gradient.
- Change in density from position 1 to position 2→ change in vorticity → Flow generation (Taylor ID)



Quantitatively

- Total PV flux $\Gamma_q = \langle\tilde{v}_x h\rangle - \rho_s^2\langle\tilde{v}_x\nabla^2\phi\rangle$
- Adiabatic limit $\alpha \gg 1$:
  +Particle flux and vorticity flux are tightly coupled (both prop. to $1/\alpha$)
- Hydrodynamic limit $\alpha \ll 1$:
  - Particle flux proportional to $1/\sqrt{\alpha}$.
  - Residual vorticity flux proportional to $\sqrt{\alpha}$.
- PV mixing still possible without ZF formation → Particles carry PV flux

- Branching ratio changes with α!

Thoughts for Experiment

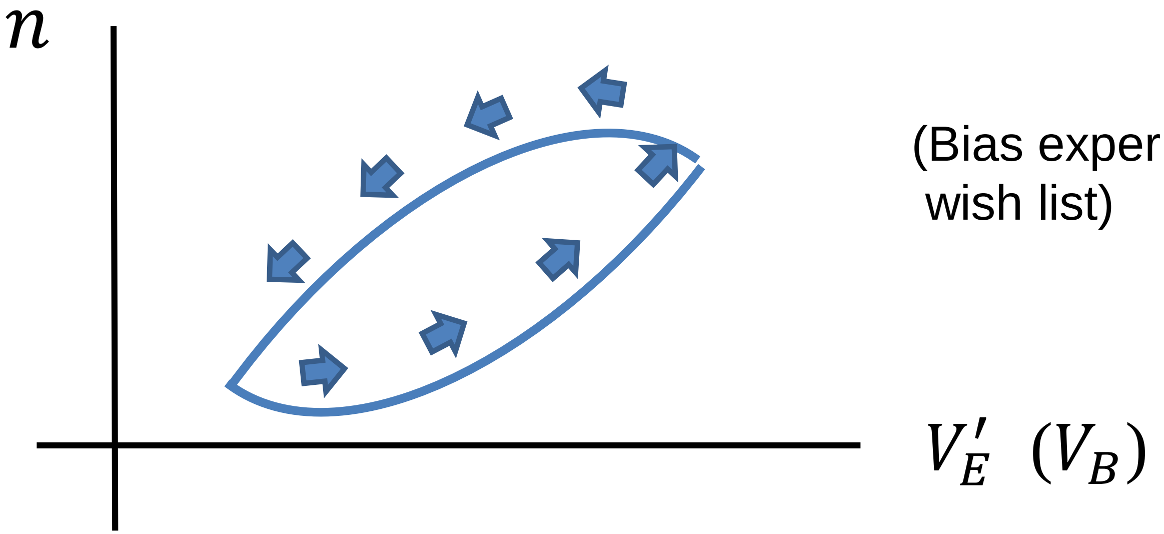
Suggestions for Experiment

- Criticality $k_{\parallel}^2 V_{The}^2/\omega\nu_e$ → T_e^2/n_e trade off
- Scale of shear layer collapse? - ρ_θ?
- Turbulence spreading penetration depth? – influence length
- Perturbative experiments: (J-TEXT, planned)
  - SMBI probe of relaxation (with fluctuations) → relaxation time
  - ExB flow drive (Bias) → enhance shear layer persistence beyond $\bar{n}_g$?
  - RMP → accelerate shear layer collapse?

N.B. Studies of turbulence and transport as $n \rightarrow n_g$, are part of (important) 'disruption question'.

In Particular:

- Can edge biasing (ala' driven L→H) sustain $\bar{n} > \bar{n}_g$ by driving shear layer?
- Is shear layer collapse hysteretic?



(Bias experiment, wish list)

- Is shear layer collapse yet another case of a back-transition of transport bifurcation?

What of H-mode?

- H-mode density limit involves back-transition prior to $\bar{n}_g$, so key

  HDL problem is high density back-transition (H→L)

- I_{turb} in SOL can exceed that of pedestal

∴

- Is HDL due
  - Shear layer or well weakening? – How?
  - Invasion of pedestal from SOL turbulence
- Coupled pedestal-SOL model under consideration

Partial Conclusions (L-mode)

- 'Density limit' is consequence of particle transport dynamics, edge cooling, etc. secondary.
- Degraded particle confinement – shear layer collapse, breakdown of self-regulation; 'Inverse' of L→H transition
- Physics: Drop in shear flow production

  Key parameter: $k_{\parallel}^2 V_{The}^2 / \omega \nu_e$ (adiabaticity)
- Penetration of turbulence spreading drives cooling front, related to MARFE etc.

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